Critical Point  Equations as icons

Why is it that particular equations, formulas and expressions become icons, asks Robert P Crease

When the 14-year-old Richard Feynman first encountered $e^{i\pi} + 1 = 0$, the future physics Nobel laureate wrote in big, bold letters in his diary that it was “the most remarkable formula in math”. Stanford University mathematics professor Keith Devlin claims that “like a Shakespearean sonnet that captures the very essence of love, or a painting that brings out the beauty of the human form that is far more than just skin deep, Euler’s equation reaches down into the very depths of existence”. Meanwhile Paul Nahin – a retired US electrical engineer – says in his recent book, Dr Euler’s Fabulous Formula, that the expression sets “the gold standard for mathematical beauty”.

For some people this expression, named after the 18th-century Swiss mathematician Leonhard Euler, even seems to have become an icon, having special significance apart from its mathematical context. It once even served as a piece of evidence in a criminal trial. In August 2003 an eco-terrorist assault on several car dealerships in the Los Angeles area resulted in millions of dollars worth of damage when a building was set alight and over 100 vehicles were destroyed or defaced. The vandalism included graffiti on the cars that read “gas guzzler” and “killer” – and, on one Mitsubishi Montero, $e^{i\pi} + 1 = 0$. Using this as a clue and later as evidence, the FBI arrested William Cottrell, a graduate student in theoretical physics at the California Institute of Technology, who was later tried and convicted. Cottrell testified at his trial that “Everyone should know Euler’s theorem”.

Icons, legitimate and illegitimate

The only equation that most people actually do know is another icon, $E = mc^2$. Einstein’s famous equation has appeared in countless movies, pop songs and cartoons. Those of a certain age, for instance, may remember the hit single Einstein A Go-Go by 1980s electronic pop band Landscape, the lyrics of which went “You’d better watch out, you’d better beware, coz Albert says that E equals mc squared”. More recently, during last year’s World Cup, the six large outdoor sculptures that were erected in Berlin to illustrate Germany’s status as the “land of ideas” included a car, a pair of football boots and a gigantic representation of $E = mc^2$.

Icons can have a dark side, when they call more attention to their image than to what they stand for. The spell cast by equations can tempt us to think that all knowledge can and ought to be couched in the form of equations, with neat packages, balanced amounts and simple units. Equations have, for example, been composed for making perfect sandwiches, workable relationships and successful sitcoms. These are, however, illegitimate attempts to create algorithms for things that cannot be quantified.

The spell of equations that I want to discuss is something different, that of genuine equations that enthral authentic scientists. In addition to the two I mentioned already, other equations that I think are legitimate icons include Maxwell’s equations – “Was it a God who wrote these signs?” Boltzmann wrote about them, quoting Goethe’s Faust – and Einstein’s equations of general relativity.

But how can an equation become an icon? After all, an equation is merely one step in the ongoing process of scientific inquiry. Euler’s expression, for example, was but one implication of his exploration of functions, while $E = mc^2$ was an afterthought of Einstein’s development of special relativity. If equations are only a means to an end, of less intrinsic value and interest than the tasks they were developed to help us with, why do some of them seem to possess an inherent value or significance beyond the process of inquiry to which they belong? Why can an abstract object like an equation literally stand alongside a pair of boots or a car?

The answer is that some steps in an inquiry acquire, and deserve, special status. Certain expressions serve as landmarks in the vital and bustling metropolis of science, a city that is continually undergoing construction and renovation. They preserve the work of the past, orient the present, and point to the future. Theories, equipment and people may change, but formulas and equations remain pretty much the same. They are guides for getting things done, tools for letting us design new instruments, and repositories for specialists to report and describe new discoveries. They summarize and store, anticipate and open up.

But there is still more to equations. As Devlin once wrote of Euler’s equation, it reaches into the depths of existence because “it brings together mental abstractions having their origins in very different aspects of our lives, reminding us once again that things that connect and bind together are ultimately more important, more valuable, and more beautiful than things that separate”.

The critical point

Devlin’s remark suggests, I think, the chief reason why equations such as those of Euler and Einstein attract value and interest beyond the particular scientific inquiries that gave birth to them. They serve as clear and concise examples of what equations and formulas do: they show how seemingly disparate elements are implicated in a unity, and do so concisely, with few moving parts, so to speak. They bring what equations do out into the open. They are like a really good joke the economy of which reveals the structure of a joke, or a proof so concise that it demonstrates what a proof is.

If equations have a dark side, it is that they can also lead us to think that knowledge resides in the equation itself, rather than in the ongoing processes of construction and renovation. They can promote the erroneous view that science consists of a set of facts or beliefs to be memorized, rather than a quest for greater understanding that is achieved by moving beyond existing facts or beliefs to new ones that we suspect are on the horizon.

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